RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, SEPTEMBER 2020

FIRST YEAR [BATCH 2019-22]

Date : 25/09/2020 **PHYSICS (Honours)**

Time : 11.00 am – 3.00 pm **Paper : III [CC3] & IV [CC4]** Full Marks : 25+25

Paper : III [CC3]

Answer **any five** questions from question nos. 1 to 8: [5×5]

1. Find the fourier series for the periodic function defined by $f(x)=e^x, -\pi \le x < \pi$. Using the series show that

$$\sum_{1}^{\infty}\frac{(-1)^n}{n^2+1}=\frac{1}{2}\left[\frac{\pi}{\sinh\pi}-1\right]$$ [3+2]

2. A function is given by

$$f(x)=\begin{cases}1\, for\, |x|<1\\ 0\, for\, |x|>1\end{cases}$$

i) Find the fourier transform and ii) show that

$$\int_0^{\infty}\frac{\sin k\cos kx}{k}dk=\begin{cases}0 & |x|>1\\ \pi/4 & |x|=1\\ \pi/2 & |x|<1\end{cases}$$

[2+3]

3. The sinusoidal wave is given by

$$f(t)=\begin{cases}V\sin wt\; 0\le t\le \dfrac{T}{2}\\ 0 \qquad \dfrac{T}{2}\le t\le T^2\end{cases}$$

Find (i) the series of the function and (ii) amplitude ratio[three ratio only]. [4+1]

4. a) Evaluate $I=\int_{-\infty}^{\infty}e^{-\alpha x^2+\beta x}dx$. [2]

b) Evaluate $\int_0^1\frac{dx}{\sqrt{1+x^4}}$. [2]

c) Show that $\mathrm{erf}(ix)=i\,erfi(x)$, where $erfi(x)$ is the imaginary error function. [1]

5. a) Prove the recursion relation: $lP_l(x)=(2l-1)xP_{l-1}(x)-(l-1)P_{l-2}(x)$. [2.5]

b) Hence show that:. $\int_{-1}^{1}x^2P_{l+1}(x)P_{l-1}(x)dx=\frac{2l(l+1)}{(2l-1)(2l+1)(2l+3)}$ [2.5]

6. a) Prove the recursion relation: $\frac{d}{dx}[x^nJ_n(x)]=x^nJ_{n-1}(x)$. [2.5]

b) Hence evaluate the integral $\int x^4J_1(x)dx$ in terms of Bessel's functions. [2.5]

7. Find the steady state potential distribution in a semi infinite plate if the bottom edge of width 30 is held at $\varphi = \begin{cases} x, & 0 < x < 15 \\ 15 - x, & 15 < \ \ x \geq 30 \end{cases}$

and the other sides are at 0. Assume there is no source or sink of charge inside the plate. [5]

8. Suppose a light string of length l is subjected to the following conditions:

$$y(0,t) = 0, y(l,t) = 0, \left.\frac{\partial y}{\partial t}\right|_{t=0} = 0.$$

Calculate the first three non-zero terms of the solution $y(x,t)$ if

$$y(x,0) = \begin{cases} kx, & 0 < x < \frac{l}{2} \\ k(l-x), & \frac{l}{2} < x < l \end{cases}$$ [5]

<u>Paper : IV [CC4]</u>

Answer **<u>any five</u>** questions from question nos. 9 to 16: [5×5]

9. a) Set up differential equation of wave. [3]

b) Whether $y = e\dfrac{(vt-z)^2}{b^2}$ b = constant, represents a wave not not. [2]

10.Let, $y = exp\left[\left(-az^2 - bt^2 - 2\sqrt{ab}zt\right)\right]$ in a wave

i) In which direction the wave propagating?

ii) What is the wave speed?

iii) Sketch the wave for time t=0 and for time t =3 sec. where $\begin{matrix} a = 144/cm^2 \\ b = 9/sec^2 \end{matrix}$ [1+1+3]

11.a) How beats form? Represent g' analytically and graphically. [1+1+1]

b) Two mutually perpendicular S.H.M are superimposed. [2]

$$x = a\cos wt$$
$$y = a\sin(wt + \pi)$$

Find its superposition state and direction.

12.a) Prove group velocity represents the particle velocity of object. [3]

b) For a mode of wave guide

$$k = \frac{w}{c}\left(1 - \frac{w_c{}^2}{w^2}\right)^{1/2}$$

Here C is the velocity of light w_c is a constant cut off frequency and other symbols are conventional. What will be the relation between group and phase velocity? [2]

13.a) If ψ_1 and ψ_2 are two solutions of the differential wave equation then show that $(\psi_1 + \psi_2)$ and $(\partial\psi_1/\partial t)$ are also solutions of the equation. [3]

b) State Huygens' principle. From this principle derive Fermat's principle. [1+1]

14.a) Consider a double slit experiment with a light containing two wavelengths 450 nm and 600 nm. Find the least order at which a maximum of one wavelength falls exactly on a minimum of the other. [2]

b) Newton's rings are formed with a source of light containing two wavelengths λ_1 and λ_2. If m^{th} order dark ring due to λ_1 coincides with the $(m+1)^{th}$ order dark ring due to λ_2, then prove that the radius of the m^{th} order dark ring of λ_1 is equal to $\left(\frac{\lambda_1 \lambda_2 R}{\lambda_1 - \lambda_2}\right)^{1/2}$, where R is the radius of curvature of the lower curved surface. [3]

15.a) In Michelson interferometer the initial and final screw readings are 10.7347 and 10.7051 as 100 fringes pass the field of view. Find the wavelength of light. [1.5]

b) Compare Fizeau fringes and Haidinger fringes. [1.5]

c) Show that the amplitude due to a large plane wavefront is just half that due to the first half-period zone acting alone. [2]

16.a) From the theory of Fresnel type diffraction with monochromatic light by a thin wire, determine the diameter of the wire. [3]

b) A zone plate is designed to bring a parallel beam of light of wavelength 600 nm to the first focus at a distance of 2 m. Calculate the radius of the central element of the zone plate. [2]

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